Universal dynamic structure factor of a strongly correlated Fermi gas

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Abstract. - Universality of strongly interacting fermions is a topic of great interest in diverse fields. Here we investigate theoretically the universal dynamic density response of resonantly interacting ultracold Fermi atoms in the limit of either high temperature or large frequency: (1) At high temperature, we use quantum virial expansion to derive universal, non-perturbative expansion functions of dynamic structure factor; (2) At large momentum, we identify that the second-order expansion function gives the Wilson coefficient used in the operator expansion product method. The dynamic structure factor is therefore determined by its second-order expansion function with an overall normalization factor given by Tan's contact parameter. We show that the spin parallel and antiparallel dynamic structure factors have respectively a tail of the form $\sim \pm\omega^{-5/2}$ for $\omega \to \infty$, decaying slower than the total dynamic structure factor found previously ($\sim \omega^{-7/2}$). Our predictions for dynamic structure factor at high temperature or large frequency are testable using Bragg spectroscopy for ultracold atomic Fermi gases.

Introduction. – The study of strongly interacting fermions has brought together very different areas of physics - neutron stars, quark-gluon plasmas, high temperature superconductors, and cold atoms - which, at first sight, have little in common. There is, however, an important generic idea of fermionic universality behind [1]: all strongly interacting, dilute Fermi gases should behave identically, depending only on scaling factors equal to the average particle separation and/or thermal wavelength, but not on the details of the interaction. Recent manipulation of ultracold Fermi gases of ^{6}Li and ^{40}K atoms near a broad collisional (Feshbach) resonance provides an ideal avenue to understand this fermionic universality [2–5]. To date, universal thermodynamics of strongly interacting fermions has been clearly demonstrated [6], by measuring the static equation of state. The purpose of this Letter is to show that universality appears in *dynamical* properties as well. We derive *exact* results for universal dynamic structure factor (DSF) at high temperatures or at large momenta and frequencies.

Exact results are very valuable for strongly interacting fermions, due to their non-perturbative, strongly correlated nature. There is no small interaction parameter to control the accuracy of theories. In specific cases, ab-initio calculations are possible using Monte Carlo methods [7–9]. However, in general this approach suffers from the fermion sign problem [10]. In 2008, Tan derived a set of exact, universal relations for two-component (spin-1/2) Fermi gases with a large s-wave scattering length a [11]. These universal relations all involve a many-body parameter called the *contact* $\mathcal{I}$, which measures the density of pairs within short distances. Tan's relations can be understood using the short-distance and/or short-time operator production expansion (OPE) method [12–14], which separates in a natural way few-body from many-body physics. For the "interaction" DSF, $\Delta S_{\sigma\sigma'}(\mathbf{q},\omega,T) \equiv S_{\sigma\sigma'}(\mathbf{q},\omega,T) - S^{(1)}_{\sigma\sigma'}(\mathbf{q},\omega,T)$, the OPE predicts that ($\mathbf{q} \to \infty$ and $\omega \to \infty$) [13,14]

$$\Delta S_{\sigma\sigma'}(\mathbf{q},\omega,T) \simeq W_{\sigma\sigma'}(\mathbf{q},\omega,T)\,\mathcal{I}, \tag{1}$$

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where $(\sigma, \sigma') =\uparrow, \downarrow$ denote the spin, $S_{\sigma\sigma'}$ and $S^{(1)}_{\sigma\sigma'}$ are respectively the DSF of interacting and non-interacting Fermi gases at the same chemical potential μ and temperature T, and $W_{\sigma\sigma'}$ are the temperature-dependent Wilson coefficients that rely only on few-body physics. At high temperatures, quantum virial expansion provides another rigorous means to bridge few-body and many-body physics [15–18]. It was shown that static thermodynamic properties of a strongly correlated Fermi gas can be expanded non-perturbatively in fugacity using some universal, temperature-independent virial coefficients [5, 17, 18], which are exactly calculable from few-fermion solutions. Both OPE and virial expansion give useful insight into the challenging many-body problem. However, their connection is yet to be understood.

In this Letter, we investigate theoretically the universal dynamic properties of a strongly correlated Fermi gas in the limit of either high temperature or large momentum/frequency. In the former limit, we show that the dynamic structure factor can be virial expanded in fugactiy, using some universal, temperature-implicit virial expansion functions. We derive, for the first time, these universal virial expansion functions for spin parallel and antiparallel DSFs of a *homogeneous* Fermi gas in the resonance (unitarity) limit, where the scattering length diverges ($a \to \pm\infty$). In the latter limit of large momentum, we show that the Wilson coefficient in Eq. (1) is given by the second-order expansion function. Therefore, the large momentum DSF is universally determined by the second order virial expansion, together with a many-body prefactor - the contact. Our results can be easily examined using Bragg spectroscopy for ultracold Fermi gases of ^{6}Li or ^{40}K atoms [19].

Universal expansion function of DSF. – Virial expansion is a powerful tool for studying the high-temperature properties of ultracold atomic Fermi gases [17]. It expresses any physical quantities of interest as an expansion in fugacity with some coefficients or functions, since the fugacity $z \equiv \exp(\mu/k_BT) \ll 1$ is a controllable small parameter at high temperatures. For the interaction DSF, the expansion is given by [20]

$$\Delta S_{\sigma\sigma'}(\mathbf{q}, \omega, T) = z^2 \Delta S_{\sigma\sigma',2} + z^3 \Delta S_{\sigma\sigma',3} + \cdots, \qquad (2)$$

where $\Delta S_{\sigma\sigma',n}(\mathbf{q}, \omega, T)$ $(n = 2, 3, \cdots)$ is the n-th expansion function. The expansion is non-perturbative as the few-body problem could be solved exactly, no matter how strong the interaction. Virial expansion is anticipated to work for temperatures down to the superfluid transition, although a nontrivial resummation of expansion terms may be required if $z \gg 1$. In Ref. [20], the lowest second order expansion function $\Delta S^{(Trap)}_{\sigma\sigma',2}$ for a *trapped* Fermi gas was calculated using two-fermion solutions in traps. However, the universal aspect of expansion functions was not realized. As a result, for different temperatures/momenta the expansion functions had to be re-calculated.

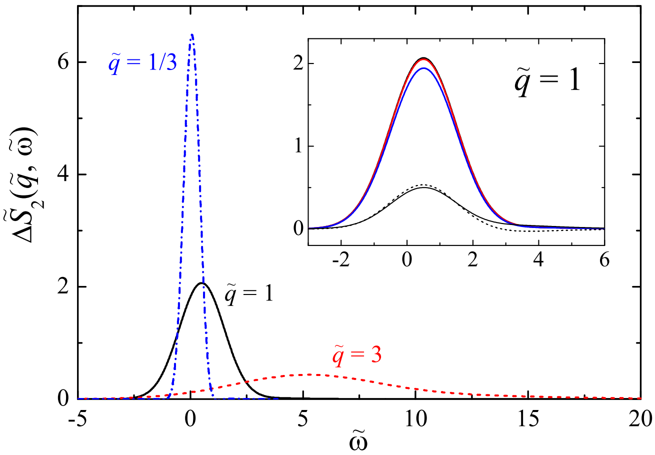

Fig. 1: (Color on-line) Universal second order expansion function of DSF at $\tilde{q} = 1/3$, 1, and 3. The inset shows the rapid convergence of $\Delta\tilde{S}_2(\tilde{q}, \tilde{\omega})$ at small $\hbar\omega_0/(k_BT)$ (thick lines) and, $\Delta\tilde{S}_{\uparrow\uparrow,2}$ (thin solid line) and $\Delta\tilde{S}_{\uparrow\downarrow,2}$ (thin dashed line) at $\tilde{q} = 1$.

Here we consider the expansion functions of a *homogeneous* Fermi gas in the unitarity limit and emphasize their universal aspect, which is not known so far. As the scattering length diverges, all microscopic scales of the interaction are absent [1]. For this few-body problem, the only energy scale is k_BT and length scale is the thermal wavelength $\lambda \equiv h/(2\pi mk_BT)^{1/2}$. Dimensional analysis leads to,

$$\Delta S_{\sigma\sigma',n}(\mathbf{q}, \omega, T) = \frac{V}{k_BT\lambda^3}\Delta\tilde{S}_{\sigma\sigma',n}(\tilde{q}, \tilde{\omega}), \qquad (3)$$

where V is the volume, $\tilde{q} = [\hbar^2\mathbf{q}^2/(2mk_BT)]^{1/2}$, $\tilde{\omega} = \hbar\omega/(k_BT)$, and $\Delta\tilde{S}_{\sigma\sigma',n}$ is a dimensionless expansion function. The temperature T is now implicit in the variables $\tilde{q}$ and $\tilde{\omega}$. This universal form implies a simple relation between the trapped and homogeneous expansion function. In a shallow harmonic trap, $V_{Trap}(\mathbf{r}) = m(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2)/2$, where $\omega_0 \equiv (\omega_x\omega_y\omega_z)^{1/3} \to 0$, the system may be viewed as a collection of many cells with a local chemical potential $\mu(\mathbf{r}) = \mu - V_{Trap}(\mathbf{r})$ and fugacity $z(r) = z\exp[-V_{Trap}(\mathbf{r})/k_BT]$, so that the trapped DSF is given by $\Delta S^{(Trap)}_{\sigma\sigma'}(\mathbf{q}, \omega, T) = \int d\mathbf{r}[\Delta S_{\sigma\sigma'}(\mathbf{q}, \omega, T, \mathbf{r})/V]$. Owing to the universal $\tilde{q}$- and $\tilde{\omega}$-dependence in the expansion functions, the spatial integration can be easily performed, giving rise to

$$\Delta\tilde{S}_{\sigma\sigma',n}(\tilde{q}, \tilde{\omega}) = n^{3/2}\frac{(\hbar\omega_0)^3}{(k_BT)^2}\Delta S^{(Trap)}_{\sigma\sigma',n}(\mathbf{q}, \omega, T). \qquad (4)$$

The (non-universal) correction to the above local density approximation is at the order of $O[(\hbar\omega_0)^2/(k_BT)^2]$. Eq. (4) is vitally important because the calculation of expansion functions in harmonic traps is much easier than in free space.

Fig. 1 reports the homogeneous expansion function $\Delta\tilde{S}_2 = 2[\Delta\tilde{S}_{\uparrow\uparrow,2} + \Delta\tilde{S}_{\uparrow\downarrow,2}]$ at three different momenta,

using $\Delta S^{(Trap)}_{\sigma\sigma',2}$ in Ref. [20] as the input. One observes a quasielastic peak at $\tilde{\omega} = \tilde{q}^2/2$ or $\omega = \hbar\mathbf{q}^2/(4m)$, as a result of the formation of fermionic pairs. We note that the third expansion function $\Delta\tilde{S}_{\sigma\sigma',3}$ or $\Delta S^{(Trap)}_{\sigma\sigma',3}$ can also be calculated straightforwardly using exact three-fermion solutions [22].

We may derive sum rules that constrain the universal expansion functions, using the well-known f-sum rules satisfied by DSF: $\int_{-\infty}^{+\infty} \omega S_{\uparrow\uparrow}(\mathbf{q},\omega,T)d\omega = N\mathbf{q}^2/(4m)$ [23] and $\int_{-\infty}^{+\infty} \omega S_{\uparrow\downarrow}(\mathbf{q},\omega,T)d\omega = 0$ [24]. The latter immediately leads to

$$\int_{-\infty}^{+\infty} \tilde{\omega}\Delta\tilde{S}_{\uparrow\downarrow,n}(\tilde{q},\tilde{\omega})d\tilde{\omega} = 0. \tag{5}$$

On the other hand, virial expansion of the total number of fermions N implies that

$$\int_{-\infty}^{+\infty} \tilde{\omega}\Delta\tilde{S}_{\uparrow\uparrow,n}(\tilde{q},\tilde{\omega})d\tilde{\omega} = n\tilde{q}^2\Delta b_n, \tag{6}$$

where Δb_n is the n-th virial coefficient and in the unitarity limit $\Delta b_2 = 1/\sqrt{2}$ and $\Delta b_3 \simeq -0.355$ [5,17].

At large momentum, the spin-antiparallel static structure factor satisfies the Tan relation [21], $\int S_{\uparrow\downarrow}(\mathbf{q},\omega,T)d\omega \simeq \mathcal{I}/(8\hbar q)$. This indicates a virial expansion of the contact: $\mathcal{I} = 16\pi^2 V\left[z^2c_2 + z^3c_3 + \cdots\right]/\lambda^4$, where the contact coefficients c_n are given by,

$$\Delta\tilde{S}_{\uparrow\downarrow,n}(\tilde{q} \gg 1) \equiv \int_{-\infty}^{+\infty} \Delta\tilde{S}_{\uparrow\downarrow,n}(\tilde{q},\tilde{\omega})d\tilde{\omega} = \frac{\pi^{3/2}c_n}{\tilde{q}}. \tag{7}$$

The expansion of the contact was alternatively obtained using Tan's adiabatic sweep relation [18]. In the unitarity limit, it was shown that $c_2 = 1/\pi$ and $c_3 \simeq -0.141$ [18]. In the same limit of large momentum, the spin-parallel static structure factor is nearly unity so that $\int S_{\uparrow\uparrow}(\mathbf{q},\omega,T)d\omega \simeq N/(2\hbar)$ [20,21]. This leads to

$$\Delta\tilde{S}_{\uparrow\uparrow,n}(\tilde{q} \gg 1) \equiv \int_{-\infty}^{+\infty} \Delta\tilde{S}_{\uparrow\uparrow,n}(\tilde{q},\tilde{\omega})d\tilde{\omega} = n\Delta b_n. \tag{8}$$

For the second expansion function, $\Delta\tilde{S}_{\sigma\sigma',2}$, we have checked numerically that all the above mentioned sum rules are strictly satisfied.

Wilson coefficient of DSF. – We now turn to the large momentum/frequency limits, where the OPE Eq. (1) is assumed to be applicable. It is clear from the equation that the Wilson coefficient determines entirely the DSF at large $(\mathbf{q},\omega)$ as far as the many-body contact is known.

At T=0, the Wilson coefficient $W_{\sigma\sigma'}$ can be calculated using Feynman diagrams [14] for dynamic susceptibility $\chi_{\sigma\sigma'}(\mathbf{r},\tau) = -\langle T_\tau\hat{\rho}_\sigma(\mathbf{r},\tau)\hat{\rho}_{\sigma'}(\mathbf{0},0)\rangle$, as $S_{\sigma\sigma'}(\mathbf{q},\omega) = -\operatorname{Im}\chi_{\sigma\sigma'}(\mathbf{q},\omega)/[\pi(1-e^{-\hbar\omega/k_BT})]$. In the limit of $(\mathbf{q},\omega) \to \infty$, the contributing diagrams to $\chi_{\sigma\sigma'}(\mathbf{q},\omega)$ are sketched in Fig. 2 [14]. Diagrammatically, the contact may be identified as the vertex function at short distance and time [18,25]: $\mathcal{I} = -m^2\Gamma(\mathbf{r}=\mathbf{0},\tau=0^-)/\hbar^4$. Therefore,

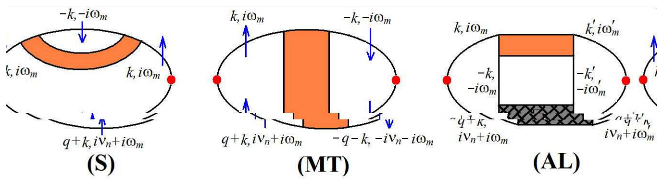

Fig. 2: (Color on-line) Diagrammatic contributions to the interaction dynamic susceptibility. The self-energy (S) and Maki-Thompson (MT) diagrams contribute to $\Delta\chi_{\uparrow\uparrow}(\mathbf{r},\tau)$ and $\Delta\chi_{\uparrow\downarrow}(\mathbf{r},\tau)$, respectively, while the Aslamazo-Larkin diagram (AL) contributes to both. The shadow in the diagrams represents the contact $\mathcal{I}$. The crossed part in the diagram (AL) is the vertex.

in the diagrams the shadow part of the vertex function $\Gamma(\mathbf{r}=\mathbf{0},\tau=0^-)$ represents the contact $\mathcal{I}$. These diagrams are well-known in condensed matter community. In the context of calculating the change in conductivity due to conducting fluctuations, the last two diagrams are called the Maki-Thompson (MT) [26] and Aslamazov-Larkin (AL) contributions [27] respectively, while first diagram gives the self-energy (S) correction. At zero temperature, we calculate these diagrams in vacuum at $\mu = 0$ and obtain that $W^{T=0}_{\uparrow\uparrow} = (f_S - f_{AL})/(4\pi^2\sqrt{m\hbar}\omega^{3/2})$ and $W^{T=0}_{\uparrow\downarrow} = (f_{MT} - f_{AL})/(4\pi^2\sqrt{m\hbar}\omega^{3/2})$, where,

$$\begin{aligned} f_S &= \frac{\sqrt{1-x/2}}{(1-x)^2}, \\ f_{MT} &= \frac{1}{\sqrt{2x}}\ln\frac{1+\sqrt{2x-x^2}}{|1-x|}, \\ f_{AL} &= \frac{1}{2x\sqrt{1-x/2}}\left[\ln^2\frac{1+\sqrt{2x-x^2}}{|1-x|} - \pi^2\Theta(x-1)\right], \end{aligned}$$

$x \equiv \hbar^2\mathbf{q}^2/(2m\hbar\omega)$, and Θ is the step function. These results agree with the previous calculations by Son and Thompson [14], although there the spin parallel and antiparallel DSFs were not treated separately. At small $\mathbf{q}^2/\omega$, we find that the spin parallel and antiparallel DSFs have the tail

$$W^{T=0}_{\uparrow\uparrow} = -W^{T=0}_{\uparrow\downarrow} = \frac{\hbar^{1/2}\mathbf{q}^2}{12\pi^2 m^{3/2}\omega^{5/2}}. \tag{9}$$

This prediction shows that for $\omega \to \infty$ the spin dependent DSFs decay an order slower in magnitude than the total DSF. The latter is proportional to $q^4/\omega^{7/2}$, as shown in Refs. [14] and [28]. The faster decay in the total dynamic structure factor is due to the cancellation of the leading terms in $W^{T=0}_{\uparrow\uparrow}$ and $W^{T=0}_{\uparrow\downarrow}$.

It is not clear how to obtain the finite temperature Wilson coefficient using diagrammatic technique, since the finite temperature diagrams involve many-body process in medium. However, Eq. (1) gives the hint. It has a strong constraint on the expansion functions of DSF. As $W_{\sigma\sigma'}$ involves only the few-body physics and hence does not

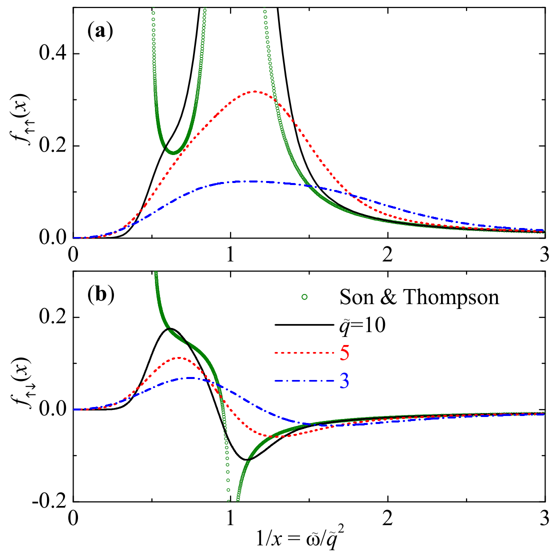

Fig. 3: (Color on-line) $f_{\sigma\sigma'} = \sqrt{m\hbar}\omega^{3/2}(z^2/\mathcal{I}_2)\Delta S_{\sigma\sigma',2}$ at $\tilde{q} = 3$, 5, and 10. With increasing momentum and/or frequency, $f_{\sigma\sigma'}$ approaches smoothly to the $T = 0$ result by Son and Thompson [14].

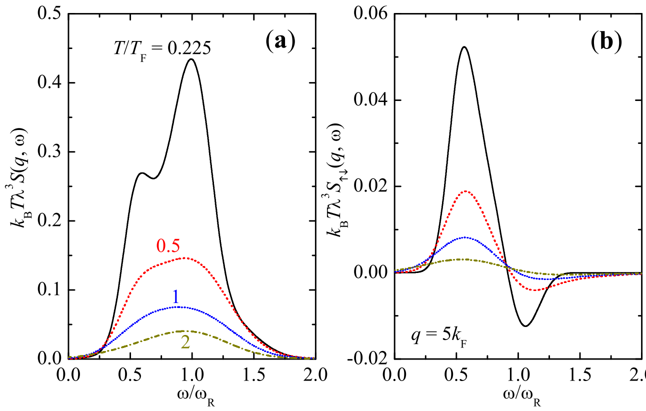

Fig. 4: (Color on-line) Universal total DSF (a) and spin-antiparallel (b) DSF at $T/T_F = 0.225$, 0.5, 1, and 2, calculated using Eq. (12). We use a pair fluctuation theory to determine the fugacity z and contact $\mathcal{I}$ [18].

contain the fugacity z, a count of the term z^n on both sides of Eq. (1) leads to

$$\Delta S_{\sigma\sigma',n}(\mathbf{q},\omega,T) = (c_n/c_2)\,\Delta S_{\sigma\sigma',2}(\mathbf{q},\omega,T), \quad (10)$$
$$W_{\sigma\sigma'}(\mathbf{q},\omega,T) = (z^2/\mathcal{I}_2)\,\Delta S_{\sigma\sigma',2}(\mathbf{q},\omega,T), \quad (11)$$

where $\mathcal{I}_2 = z^2 16\pi^2 V c_2/\lambda^4$ is the contact up to the second order expansion [18]. Therefore, the Wilson coefficient is given by the second expansion function. This result is obtained by applying the OPE and virial expansion method. As a result, in principle it should be valid at temperatures above the superfluid transition. However, we may expect that it holds at all temperatures, as both the Wilson coefficient and second expansion function are irrelevant to the many-body pairing in the superfluid phase. The many-body effect enters through the many-body parameter of contact only.

Eq. (11) is the main result of this Letter. At a first glance, it may be a suprising result. However, it could be understood from the proportionality shown in Eq. (10). On the other hand, a rigorous proof of Eq. (10) at large q and ω justifies the use of the OPE method. We note that, for the two-component Fermi gas, our result shows that the Wilson coefficient is determined by the two-body physics solely. This is in agreement with previous work. For example, in Ref. [13] a two-body scattering amplitude was used to calculate the zero temperature Wilson coefficient for the rf-spectroscopy of a strongly interacting Fermi gas. However, in general cases where three or four-body physics come into play, we anticipate that the Wilson coefficient should be related to the higher order virial expansion function. In that case, new universal relations with new many-body "contact" parameters would appear.

In Fig. 3 we check the validity of Eq. (11) at $T = 0$, by calculating $\sqrt{m\hbar}\omega^{3/2}(z^2/\mathcal{I}_2)\Delta S_{\sigma\sigma',2}$ at different momenta. With decreasing temperature T or increasing $\tilde{q} \propto q/\sqrt{T}$, it approaches gradually to $\sqrt{m\hbar}\omega^{3/2}W_{\sigma\sigma'}^{T=0}$. This confirms numerically that Eq. (11) holds at zero temperature. Moreover, at high temperatures where the fugacity is small, to a good approximation we have the interaction DSF $\Delta S_{\sigma\sigma'} \simeq z^2 \Delta S_{\sigma\sigma',2}$. As the contact $\mathcal{I} \simeq \mathcal{I}_2$ at high T, it is trivial to confirm Eq. (11). Note that, in the limit of large frequency, the tail $\omega^{-5/2}$ of $W_{\uparrow\uparrow}^{T=0}$ and $W_{\uparrow\downarrow}^{T=0}$ is fairly evident in the second order virial expansion functions.

Universal DSF at large $(\mathbf{q},\omega)$. – In this limit, using Eqs. (1) and (11) the DSF is approximated by

$$S_{\sigma\sigma'} \simeq S_{\sigma\sigma'}^{(1)}(\mathbf{q},\omega,T) + \frac{\mathcal{I}\lambda^4}{16\pi V}\Delta S_{\sigma\sigma',2}(\mathbf{q},\omega,T). \quad (12)$$

This approximate DSF should be *quantitatively* accurate for sufficiently large momentum and frequency. It holds at all temperatures and the many-body effect is respected by the contact. However, the momentum q should be larger than a critical momentum $q_c \gg max(\lambda^{-1}, k_F)$ in order to validate the use of the OPE equation (1). Here, k_F is the Fermi wavevector. Quantitatively, an estimate of q_c requires the calculation of $\Delta S_{\sigma\sigma',n>2}$ and the examination of Eq. (10). We note that Eq. (12) may not be reliable at small frequency, $\omega \sim 0$. As a result, the structure factor sum-rules may not be strictly satisfied. We note also that, in the limit of high temperatures where the contact $\mathcal{I} \simeq \mathcal{I}_2$, the prefactor of the $\Delta S_{\sigma\sigma',2}$ term is z^2. Therefore, the approximate DSF reduces back to the virial expansion up to the second order. In this high-T limit, Eq. (12) is valid for arbitrary q and ω.

We present in Fig. 4 the temperature dependence of the approximation DSF of a normal, homogeneous unitary Fermi gas at $q = 5k_F$, by assuming that $q_c \sim 5k_F$. The many-body contact and fugacity are calculated by us-

ing a strong-coupling pair fluctuation theory [18], which is shown to be accurate for describing the unitary equation of state. Close to the superfluid transition temperature, a quasielastic peak clearly emerges at the recoil energy for pairs, $\omega = \hbar\mathbf{q}^2/(4m)$, in agreement with the low-temperature experimental observation [19].

Observation of universal DSF at large frequency. – Eq. (12) indicates that at large $(\mathbf{q}, \omega)$, the interaction DSF of a unitary Fermi gas depends on the reduced moment $\tilde{q} = q\lambda/\sqrt{4\pi}$ and reduced frequency $\tilde{\omega}$ only. This universal dependence could be examined using large-momentum Bragg spectroscopy [19, 21], with varying momentum and temperature while keeping $\tilde{q}$ invariant. One can also extract experimentally the universal second expansion function $\Delta\tilde{S}_{\sigma\sigma',2}(\tilde{q}, \tilde{\omega})$ since the contact $\mathcal{I}$ can be determined independently using the f-sum rule [21]. These predictions break down below the critical momentum q_c. Note that, by tuning the transferred momentum in Bragg beams, the value of $q_c \gg max(\lambda^{-1}, k_F)$ might be determined experimentally.

Conclusion. – We have studied the finite temperature dynamic structure factor of a homogeneous unitary Fermi gas, using quantum virial expansion and operator product expansion. The universal second order expansion function has been calculated and related to the Wilson coefficient at large momentum q. As a result, in that limit the thermal wavelength λ becomes the only length scale and the interaction dynamic structure factor should depend universally on a reduced momentum $q\lambda/\sqrt{4\pi}$. We have proposed that Bragg spectroscopy with large transferred momentum should be able to confirm this universal dependence. Our results can be extended to other dynamical properties of a strongly correlated Fermi gas, such as the rf-spectrum and single-particle spectral function.

It is an interesting chanllenge to derive from three- and four-fermion solutions [22, 29] new universal relations involving a many-body parameter like Tan's contact. The determination of Wilson coefficients in that case should be difficult. Our method of calculating higher-order virial expansion function would give the most natural and convenient way to obtain the Wilson coefficient.

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